



General Certificate of Secondary Education  
2023

Centre Number

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Candidate Number

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## Further Mathematics

Unit 1 (With calculator)

Pure Mathematics



[GFM11]

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**FRIDAY 2 JUNE, MORNING**

### TIME

2 hours.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

**You must answer the questions in the spaces provided.**

**Do not write outside the boxed area on each page.**

Complete in black ink only. **Do not write with a gel pen.**

All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.

Answer **all fourteen** questions.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 100.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on page 2.

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## Formula Sheet

### PURE MATHEMATICS

Quadratic equations: If  $ax^2 + bx + c = 0$  ( $a \neq 0$ )

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Differentiation: If  $y = ax^n$  then  $\frac{dy}{dx} = nax^{n-1}$

Integration:  $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$  ( $n \neq -1$ )

Logarithms: If  $a^x = n$  then  $x = \log_a n$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices:

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{then } \det \mathbf{A} = ad - bc$$

$$\text{and } \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (ad - bc \neq 0)$$



1 Find  $\frac{dy}{dx}$  if  $y = \frac{2}{3}x^6 + \frac{1}{2x^4}$

Answer \_\_\_\_\_ [2]



2 Express  $y$  in terms of  $x$  if

$$\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 4$$

given that  $y = 7$  when  $x = 2$

Answer  $y =$  \_\_\_\_\_ [5]





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**(Questions continue overleaf)**

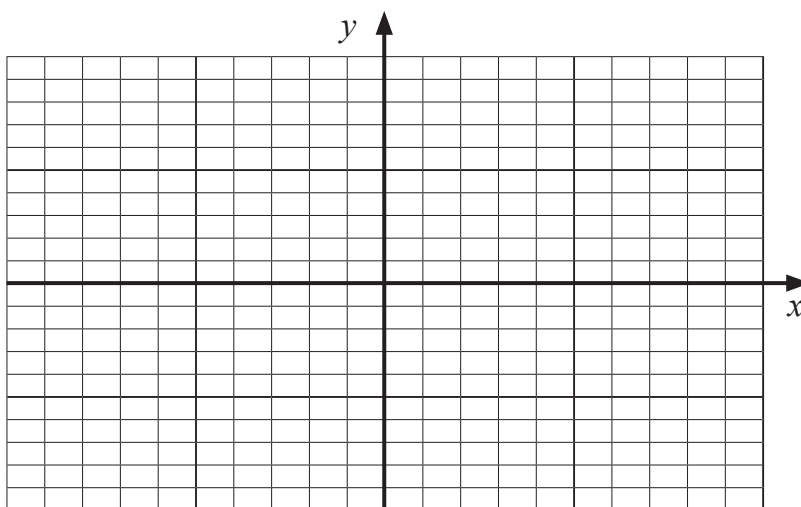
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- 3 (a) Sketch the graph of  $y = \cos x$  for  $-180^\circ \leq x \leq 180^\circ$  on the axes below.



[2]

- (b) (i) Solve the equation

$$3 \cos x = -2$$

$$\text{for } -180^\circ \leq x \leq 180^\circ$$

Answers \_\_\_\_\_ [3]



(ii) Hence solve the equation

$$3 \cos\left(\frac{\theta}{2} - 10^\circ\right) = -2$$

for  $-360^\circ \leq \theta \leq 360^\circ$

Answers \_\_\_\_\_ [3]

[Turn over



4 A function  $f(x)$  is defined by

$$f(x) = x^2 - 8x + 24$$

(i) Use the method of **completing the square** to rewrite  $f(x)$  in the form

$$(x - a)^2 + b$$

where  $a$  and  $b$  are constants.

Answer \_\_\_\_\_ [2]



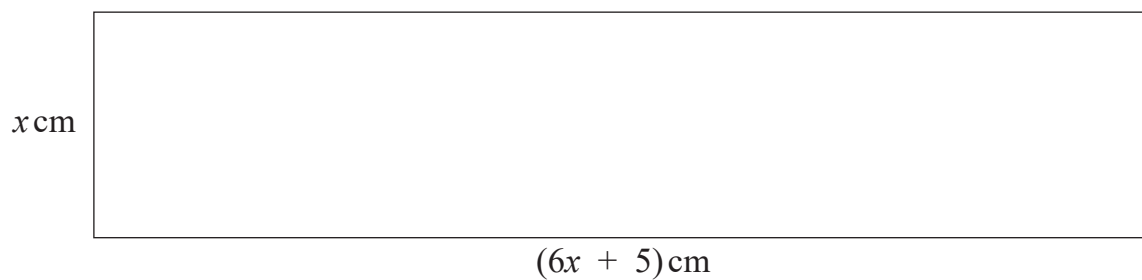
(ii) Hence write down the coordinates of the minimum turning point of the curve

$$y = x^2 - 8x + 24$$

Answer \_\_\_\_\_ [2]



- 5 A rectangle has width  $x$  cm and length  $(6x + 5)$  cm, as shown below.



Find the range of values of  $x$  for which the area of the rectangle is less than  $6 \text{ cm}^2$

You **must** show clearly each stage of your solution.

Answer \_\_\_\_\_ [6]



6 Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 5 & -2 \\ 4 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

(i) Solve the matrix equation  $\mathbf{AX} = \mathbf{C}$

Answer \_\_\_\_\_ [4]

(ii) Explain why it is impossible to solve the equation  $\mathbf{BX} = \mathbf{C}$

Answer \_\_\_\_\_  
\_\_\_\_\_ [1]

[Turn over



7 (a) If  $\log_7(3) = a$  and  $\log_7(2) = b$   
express  $\log_7(10.5)$  in terms of  $a$  and  $b$ .

Answer \_\_\_\_\_ [2]



(b) Solve the equation

$$9\left(\frac{2}{5}x - 2\right) = 4$$

Answer \_\_\_\_\_ [4]

[Turn over

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8 Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad \mathbf{B} = [3 \quad -1] \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

Calculate

(i) **AB**

Answer \_\_\_\_\_ [2]



(ii)  $C^2 - AB$

Answer \_\_\_\_\_ [3]

[Turn over

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9 Simplify **fully** the following algebraic expression

$$\left(\frac{x}{2} - \frac{2}{x}\right) \div \frac{(x+2)}{6x}$$

Answer \_\_\_\_\_ [4]



10 A curve is defined by the equation  $y = 2x^3 - 3x^2 + x + 5$

Find the equation of the **normal** to the curve at the point P  $(-1, -1)$ .

Answer \_\_\_\_\_ [7]

[Turn over

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11 Jack, Jill and Jared individually deposited money in each of three banks, A, B and C.

Bank A pays  $x\%$  interest each year.

Bank B pays  $y\%$  interest each year.

Bank C pays  $z\%$  interest each year.

Jack deposited £1500, £3500 and £4500 in banks A, B and C respectively, and received £305 overall interest in a year.

(i) Show that  $x$ ,  $y$  and  $z$  satisfy the equation

$$3x + 7y + 9z = 61$$

[1]



Jill deposited £1000, £2000 and £6000 in banks A, B and C respectively, and received £330 overall interest in a year.

(ii) Show that  $x$ ,  $y$  and  $z$  also satisfy the equation

$$x + 2y + 6z = 33$$

[1]

Jared deposited £2000, £3000 and £5000 in banks A, B and C respectively, and received £320 overall interest in a year.

(iii) Show that  $x$ ,  $y$  and  $z$  also satisfy the equation

$$2x + 3y + 5z = 32$$

[1]

[Turn over



(iv) Solve the equations below to find the percentage interest per year paid by each of the banks, showing clearly each stage of your solution.

$$3x + 7y + 9z = 61$$

$$x + 2y + 6z = 33$$

$$2x + 3y + 5z = 32$$





Answer Bank A \_\_\_\_\_ %

Bank B \_\_\_\_\_ %

Bank C \_\_\_\_\_ % [8]

[Turn over

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12 A curve is defined by the equation  $y = (2x - 1)(x - 2)^2$

(i) Write down the **coordinates** of the points where the curve meets the  $x$ -axis.

Answer \_\_\_\_\_ [2]

(ii) Write down the **coordinates** of the point where the curve meets the  $y$ -axis.

Answer \_\_\_\_\_ [1]



(iii) Expand fully the expression  $(2x - 1)(x - 2)^2$

Answer \_\_\_\_\_ [3]



(iv) Find the coordinates of the turning points of the curve.

Answer \_\_\_\_\_ [6]



- (v) Using calculus, identify each turning point as either a maximum or a minimum point.

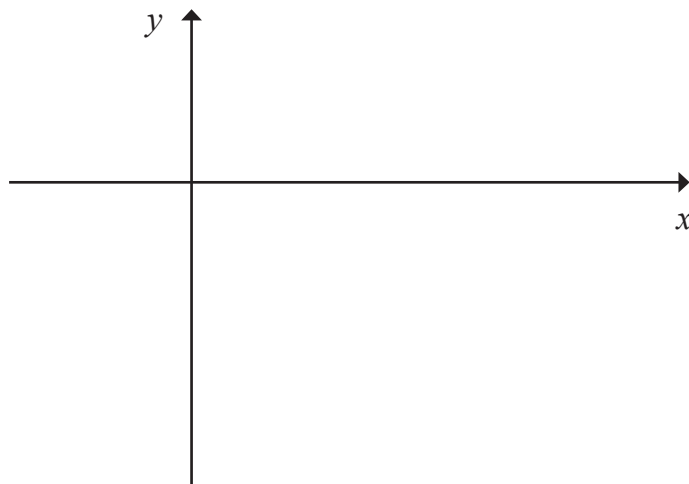
You **must** show working to justify your answer.

Answer \_\_\_\_\_ [2]

- (vi) Hence, using your answers from parts (i) to (v), sketch the curve of

$$y = (2x - 1)(x - 2)^2$$

on the axes below.



[3]

[Turn over



- 13 John took out an investment 20 years ago. Over the next 20 years he recorded the value of his investment at various times.

The table below shows the value of his investment,  $V$ , in pounds, after  $T$  years.

Years $T$	Value $V$ (£)		
3	673.14		
5	715.70		
8	757.22		
14	809.18		
20	845.23		

It is believed that a relationship of the form

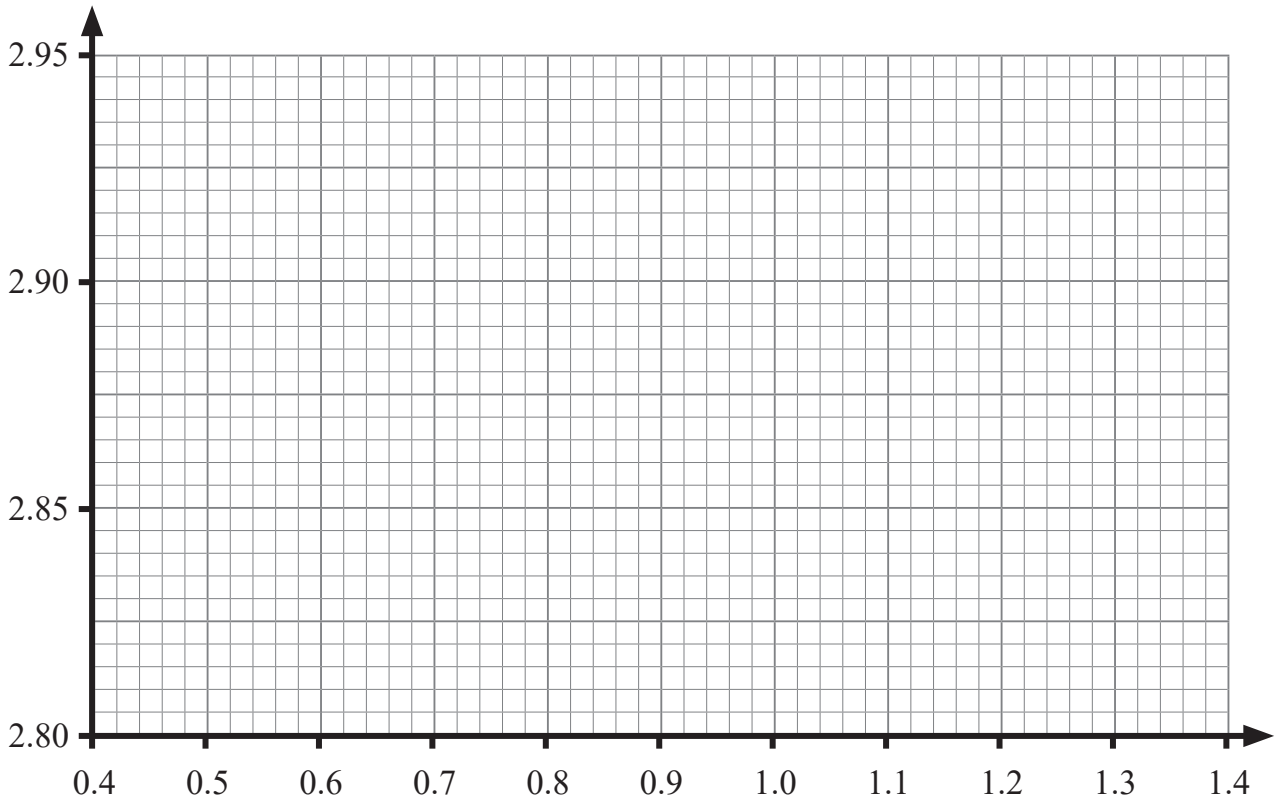
$$V = aT^n$$

exists between years 3 and 20, where  $a$  and  $n$  are constants.

- (i) Verify that a relationship of the form  $V = aT^n$  exists by drawing a suitable straight line graph on the grid opposite.

Show clearly the values used, correct to 3 decimal places, in the table above.





[6]

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[Turn over



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(ii) Hence, find the values of  $a$  and  $n$ , giving  $a$  correct to the nearest integer and  $n$  correct to 2 decimal places.

Answer  $a =$  \_\_\_\_\_ ,  $n =$  \_\_\_\_\_ [4]



(iii) Use the formula  $V = aT^n$ , with your values for  $a$  and  $n$ , to calculate after how many years the value of the investment was £830

Give your answer correct to 1 decimal place.

You **must** show your working,

Answer \_\_\_\_\_ years [2]

[Turn over

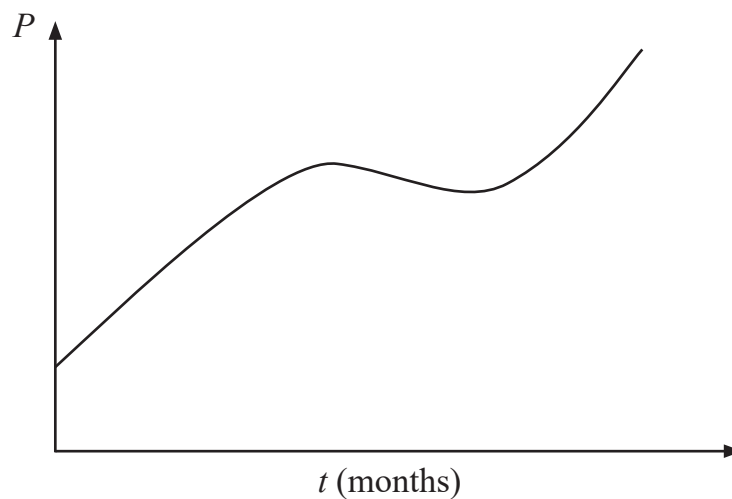


14 On a remote island the population  $P$  of lizards is modelled by the function

$$P = t^3 - 12t^2 + 45t + 10$$

where  $t$  is the time in months since the counting of lizards began.

A sketch of the population of lizards against time in months is shown below.



(i) Write down the initial population of lizards when counting began.

Answer \_\_\_\_\_ [1]



(ii) Using calculus, find the time, in months, after which the population **started to fall**.

Answer \_\_\_\_\_ [5]

Q14 continues on page 33

[Turn over

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(iii) What was the minimum number of lizards after the population started to fall, before it began to rise again?

Answer \_\_\_\_\_ [2]

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<b>Total Marks</b>	
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Examiner Number

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